

Ultra-high energy particle collisions in a regular spacetime without blackholes or naked singularities

Mandar Patil ^{*} and Pankaj S. Joshi [†]
Tata Institute of Fundamental Research
Homi Bhabha Road, Mumbai 400005, India

We investigate here the particle acceleration and collisions with extremely large center of mass energies in a perfectly regular spacetime containing neither singularity nor an event horizon. The ultra-high energy collisions of particles near the event horizon of extremal Kerr blackhole, and also in many other examples of extremal blackholes have been investigated and reported recently. We studied an analogous particle acceleration process in the Kerr and Reissner-Nordstrom spacetimes without horizon, containing naked singularities. Further to this, we show here that the particle acceleration and collision process is in fact independent of blackholes and naked singularities, and can happen in a fully regular spacetime containing neither of these. We derive the conditions on the general static spherically symmetric metric for such a phenomena to happen. We show that in order to have ultra-high energy collisions it is necessary for the norm of the timelike Killing vector to admit a maximum with a vanishingly small but a negative value. This is also a condition implying the presence of a surface with extremely large but nevertheless finite value of the redshift or blueshift. Conditions to have ultrahigh energy collisions and regular center imply the violation of strong energy condition near the center while the weak energy condition is respected in the region around the center. Thus the central region is surrounded by a dark energy fluid. Both the energy conditions are respected at the location where the high energy collisions take place. As a concrete example we then investigate the acceleration process in the spacetime geometry derived by Bardeen which is sourced by a non-linear self-gravitating magnetic monopole.

PACS numbers: 04.20.Dw, 04.70.-s, 04.70.Bw

I. INTRODUCTION

Various particle accelerators like the Large Hadron Collider can accelerate and collide the particles upto 10Tev. Physics beyond this scale, all the way upto Planck scale which is at 10^{19} GeV remains unexplored experimentally. An intriguing possibility is to explore the naturally occuring astrophysical phenomena where such collisions can take place, and to extract information about the new physics from the signals that we get from such collision events.

In this direction it was suggested by Banados, Silk and West [1] that extremal or near extremal Kerr blackholes can serve the purpose. They showed that it would be possible to have collisions with extremely large center of mass energies in the vicinity of the event horizon of the extremal Kerr blackholes, even if the colliding particles start out from rest at infinity. For such collisions to occur it is necessary, however, to have the geodesic parameters of one of the colliding particles highly fine-tuned and the proper time required for the collisions in the rest frame of this particle also turns out to be infinite. Various other drawbacks of this process, when it is explored in the context of realistic astrophysical scenario were discussed in [2]. Different aspects of particle acceleration mechanism for Kerr blackhole were investigated

subsequently [3]. The emergent flux of the particles like neutrinos created in the collisions of dark matter particles in the Kerr blackhole geometry around the event horizon was computed and shown to be observable with detectors like Icecube [4]. It was also argued that the maximum center of mass energy of collision achievable, taking into account the backreaction, might be actually significantly smaller than the Planck scale [5]. This process was also shown to occur in various other extremal blackhole geometries [6].

We showed that the process of ultrahigh energy collisions goes beyond the blackhole geometries and can also be extended to the spacetimes having no event horizon but containing the naked singularities. We explored the Kerr and Reissner-Nordstrom geometries from this perspective. We showed that it is possible to have high energy collisions in the Kerr geometry with spin parameter exceeding unity by a vanishingly small amount. The event horizon is absent in this case and the singularity is exposed to the asymptotic observers. The high energy collisions take place at a location faraway from the singularity, where the horizon would have been present in the extremal Kerr blackhole case. We considered the particles that follow the geodesic motion in the equatorial plane as well as along the axis of symmetry [7]. Due to the absence of the event horizon collisions can take place between the radially ingoing and outgoing particles unlike in the blackhole case where one is forced to consider the collisions between two ingoing particles only. We showed that this allows us to avoid the extreme fine-tuning of the geodesic parameters and that the proper

^{*}Electronic address: mandarp@tifr.res.in

[†]Electronic address: psj@tifr.res.in

time required for the collisions to take place also happens to be finite. Interestingly, the repulsive effect of gravity near the naked singularity plays an important role near the axis of symmetry in the particle collision process. We also explored the Reissner-Nordstrom geometry from the perspective of the particle acceleration and performed an exact calculation computing the center of mass energy of collision taking into account the backreaction effect of the colliding particles on the background geometry. This can be done by considering the collision between the spherical shells of particles instead of individual particles themselves [8]. As we showed, the maximum center of mass energy of collision achievable can be many orders of magnitude larger than the Planck energy. We also showed that, on an average, about half of the particles which are produced in the high energy collisions would escape to infinity, essentially due to the absence of the event horizon and thus would give rise to a large flux of emergent particles. [9]. On the other hand, in the blackhole case, due to the presence of the event horizon, the emergent flux would be small as most of the particles generated in the collisions are captured by the event horizon.

We also showed that the ultrahigh energy collisions between the ingoing particles can take place in the vicinity of the Janis-Newmann-Winicor naked singularities, obtained by generalizing Schwarzschild metric by invoking a massless scalar field, while such collisions were absent in the Schwarzschild blackhole geometry[10].

In the present work, we go one step ahead and show that it would be possible to have ultrahigh collisions in the absence of both blackholes as well as naked singularities. This is quite surprising because based on the intuition it might be thought that it would be necessary to have either blackholes or singularities in order to accelerate the particles and make them collide with large center of mass energies.

For the sake of simplicity and definitiveness we focus here on the spherically symmetric static spacetimes. We derive the conditions necessary to have high energy particle collisions and at the same time for the spacetime to be perfectly regular and devoid of any singularity and event horizon. As an concrete example we discuss then the spacetime metric proposed by Bardeen [11]. It is a two-parameter solution, the parameters being the mass and the charge of the geometry. When the mass parameter is larger than a certain critical value, the spacetime contains a blackhole which has a regular center and which is interestingly free from a central singularity. When the mass parameter is smaller than a critical value the spacetime does not contain an event horizon and is still regular everywhere. It was shown that the source of the energy momentum tensor is the self-gravitating non-linear magnetic monopole [12]. We show that ultra-high energy collisions can take place in this spacetime in a generic manner without requiring any fine-tuning of the geodesic parameters and also requiring only a finite proper time for such collisions to take place.

We also investigate here the properties of the matter that might source such a geometry which has a regular center and hosts high energy collisions. We show that the conditions for the above and to have negative density gradient at the center imply the violation of strong energy condition near the center while the weak energy condition is still respected. Thus the center is surrounded by some sort of a dark energy fluid. Both the energy conditions can be shown to hold good at the point of collision. It would be interesting to investigate whether or not such a situation could arise in a realistic astrophysical scenario. Gravastars could possibly provide a scenario where such conditions could be met[13].

In the next Section II, we consider the particle acceleration in a general static spherical spacetime geometry which does not contain any event horizon. In Section III we discuss the energy conditions that must be satisfied by the matter for the regularity and for high energy collisions to occur. Section IV considers an explicit example, which is the Bardeen monopole geometry, in order to illustrate these considerations and to see their applications. We finally conclude with a discussion summarizing the implications of our results.

II. PARTICLE ACCELERATION IN GENERAL SPHERICALLY SYMMETRIC REGULAR STATIC SPACETIMES WITHOUT AN EVENT HORIZON

In this section we study the particle acceleration process in the general spherically symmetric static spacetime whose metric is given by (1). A wide variety of spacetimes admit metric of this form. We derive the conditions to have ultra-high energy particle collisions and to have a regular center in the spacetime with no event horizon. In other words, the spacetime has neither a black hole, nor a naked singularity.

We deal here with the spherically symmetric metric of the general form,

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2 \quad (1)$$

There is only one free function of the radial coordinate, namely $f(r)$, in the metric and we shall impose the conditions on it so as to have the above desired properties.

Since we are dealing with the spacetime that is static, it admits a timelike Killing vector which is given by $k = \partial_t$ and we have

$$f(r) = -k.k \quad (2)$$

This implies that the free metric function $f(r)$ is the negative of the norm of the timelike Killing vector k . Thus the conditions that we impose on the metric function $f(r)$ can be easily elevated to the coordinate independent conditions to be imposed on the timelike Killing vector.

We further assume that the spacetime is asymptotically flat. The norm of the timelike Killing vector is normalized to $k.k = -f \rightarrow -1$ at infinity as $r \rightarrow \infty$.

The condition for the existence of the event horizon at any value of r is given by $f(r) = 0$. Thus in order to avoid the horizon at any r we must have the following condition imposed on the metric function,

$$f(r) > 0 \quad (3)$$

namely, it must be nonzero and positive.

Now we derive conditions that must be imposed on the metric function $f(r)$ so as to ensure the absence of the spacetime singularity. Essentially we must ensure that the curvature invariants take a finite value everywhere. We rewrite the function $f(r)$ as

$$f(r) = 1 - \frac{2M(r)}{r} \quad (4)$$

where it has been traded for another function $M(r)$ of

the radial coordinate, which is now the Misner-Sharp mass for the system. For the Schwarzschild spacetime this function is a constant, $M(r) = M$, as a consequence of which there is an event horizon at $r = 2M$ and a singularity exists at $r = 0$. In our case we must therefore have

$$2M(r) < r \quad (5)$$

in order to avoid the horizon. We must also have $M(r)$ either tending to a constant value or increasing slower than r as we approach infinity $r \rightarrow \infty$ so as to have $f(r) \rightarrow 1$. We now derive the conditions that must be imposed on $M(r)$ in order to avoid the singularity.

Various curvature invariants can be written as

$$\begin{aligned} R &= R_{\mu\nu}g^{\mu\nu} = \frac{4M' + 2rM''}{r^2} \\ R_1 &= R_{\mu\nu}R^{\mu\nu} = \frac{8M'^2 + 2r^2M''^2}{r^4} \end{aligned} \quad (6)$$

$$K = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = \frac{4}{r^6} (12M^2 + 4rM(-4M' + rM'') + r^2(8M'^2 - 4rM'M'' + r^2M''^2))$$

All curvature invariants are finite at finite values of the radial coordinate as long as the condition (5) for the absence of the horizon is satisfied.

However, various curvature invariants could diverge at the center $r = 0$. We Taylor expand $M(r)$ around $r = 0$

$$M(r) = M_0 + M_1r + M_2r^2 + M_3r^3 + \dots \quad (7)$$

For the curvature invariants to be finite at the center, it turns out we must have

$$M_0 = M_1 = M_2 = 0 \quad (8)$$

The first nonzero coefficient in the Taylor expansion must be M_3 . Thus the metric near center can be written as

$$ds^2 = -(1 - 2M_3r^2)dt^2 + (1 + 2M_3r^2)dr^2 + r^2d\Omega^2 \quad (9)$$

If $M_3 \neq 0$ then the metric near the center looks like de Sitter.

Equivalently the metric function $f(r)$ must satisfy the following conditions,

$$f(r=0) = 1; f'(r=0) = 0 \quad (10)$$

Thus if the metric satisfies the conditions above and if $f(r)$ takes a non-zero positive value, we can then avoid singularity as well as event horizon in the spacetime.

We now derive the necessary and sufficient conditions for the occurrence of ultrahigh energy collisions in spherically symmetric static asymptotically flat spacetimes containing no singularity or horizon.

Consider a particle which is following a geodesic motion in the spherically symmetric static spacetime. Let $U^\mu = (U^t, U^r, U^\theta, U^\phi)$ be the velocity of the particle. The motion of such a particle would be confined to a plane which can be taken to be the equatorial plane ($\theta = \frac{\pi}{2}$). Thus

$$U^\theta = 0 \quad (11)$$

Spherical symmetry and the static nature of spacetime implies the existence of the Killing vectors $k = \partial_t, l = \partial_\phi$. The following quantities are then the constants of motion,

$$\begin{aligned} E &= -\partial_t.U = f(r)U^t \\ L &= \partial_\phi.U = r^2U^\phi \end{aligned} \quad (12)$$

E and L can be interpreted as the conserved energy and angular momentum per unit mass of the particle. We can thus write the components of velocity as

$$\begin{aligned} U^t &= \frac{E}{f(r)} \\ U^\phi &= \frac{L}{r^2} \end{aligned} \quad (13)$$

Using normalization condition for velocity $U.U = -1$ and the (11),(13), we can write the radial component of the velocity U^r as

$$U^r = \pm \sqrt{E^2 - f(r) \left(1 + \frac{L^2}{r^2}\right)} \quad (14)$$

Here \pm stands for the radially outgoing and ingoing particles respectively. This equation can be recast in the form

$$U^{r2} + V_{eff}(r, L) = E^2 \quad (15)$$

where

$$V_{eff}(r, L) = f(r) \left(1 + \frac{L^2}{r^2}\right) \quad (16)$$

can be thought of as an effective potential for the motion in the radial direction. The necessary condition for the particle to reach a location with a radial coordinate r is that the quantity $E^2 - V_{eff}(r, L) \geq 0$ must be non-negative, which is the condition for the radial velocity of the particle to be real. If it is nonzero then the particle reaches that point with a nonzero velocity. If it is zero then its velocity is zero and it turns back. Whereas, if it is zero and furthermore the potential also admits a maximum $V_{eff}(r, L)' = 0$ then the particle will asymptotically reach that location at an infinite proper time.

We now consider a collision between two particles. For the sake of simplicity we assume that the particles have the same mass and that they also travel in the same plane which is taken to be the equatorial plane. Let m be the mass and U^1, U^2 be the velocities of the particles. The conserved energy and angular momenta are E_1, E_2, L_1, L_2 . The center of mass energy of collision between the particles at the radial coordinate r is given by

$$\frac{E_{c.m.}^2}{2m^2} = 1 - U_1.U_2 \quad (17)$$

From (1),(11),(13),(14) we get

$$\frac{E_{c.m.}^2}{2m^2} = 1 + \frac{E_1 E_2}{f(r)} - k \frac{|L_1||L_2|}{r^2} - \kappa \frac{1}{f(r)} \sqrt{E_1^2 - f(r) \left(1 + \frac{L_1^2}{r^2}\right)} \sqrt{E_2^2 - f(r) \left(1 + \frac{L_2^2}{r^2}\right)} \quad (18)$$

where $\kappa = 1$ if either both the particles are radially ingoing or they are outgoing. However, if one of the particles is radially ingoing and the other one is radially outgoing then we have $\kappa = -1$. Whereas $k = 1$ corresponds to the case where angular momenta of both the particles have same sign and $k = -1$ corresponds to the case where angular momenta of the particles have opposite signs. We shall deal with these cases separately.

From (18) it is evident that there is a possibility that the center of mass energy of collision can possibly diverge

when either $r = 0$ or when $f(r) \rightarrow 0$ at some finite value of the radial coordinate r .

We now show that the center of mass energy of collision cannot diverge at the center of the spacetime $r = 0$. It is clear from (14) that a particle will reach the center only if its angular momentum is zero. Thus we must have $L_1 = L_2 = 0$ in order for particles under consideration to reach the center and participate in the collision. And also it follows from (13) that we must have $E_1, E_2 > 0$. This is because absence of the horizon implies $f(r) > 0$ and the velocity being future direction vector $U^t > 0$. The center of mass energy of collision between the particles at the center using the fact $f(0) = 1$ is then given by

$$\frac{E_{c.m.}^2}{2m^2} = 1 + E_1 E_2 - \kappa \sqrt{E_1^2 - 1} \sqrt{E_2^2 - 1} \quad (19)$$

which is clearly finite.

We now turn to the second case where $f(r) \rightarrow 0$ for some finite value of r . It follows from (14) that for a particle with a given energy E , the angular momentum must be in the following range for it to reach r and participate in the collision,

$$L \in \left(-r \sqrt{\frac{E^2}{f(r)} - 1}, r \sqrt{\frac{E^2}{f(r)} - 1} \right)$$

Since $f(r) \rightarrow 0$ the limiting values of the allowed angular momentum $L_{\pm} = \pm r \sqrt{\frac{E^2}{f(r)} - 1}$ are extremely large.

We shall deal here with the following cases. In the first case both the particles have finite radial velocity at the point of collision. The angular momenta of both the particles are sufficiently away from the extreme critical values of the allowed angular momentum interval. In the second case both the particles take vanishingly small value of radial component of velocity, but have nonzero angular velocity at the collision. The angular momenta of both the particles are arbitrarily close to the limiting values. In the third case the radial velocity of one of the particles is finite whereas the second particle has small radial component but finite angular velocity at the point of collision. The angular momentum of one of the particles is sufficiently away from the limiting value and that of the second particle is arbitrarily close to the limiting value. In the fourth case one of the particles has both radial as well as angular component of the velocity vanishing, whereas the second particle has either or both components of the velocity non-vanishing at the collision. In the first three cases both the particles have finite conserved energy. Whereas in the fourth case the conserved energy of the first particle takes a vanishing value, while second particle has finite conserved energy.

We note that of these, the first case is generic, whereas other cases require a fine-tuning.

Case 1

We first focus on the case where the angular momenta of the particles are well within the interval away from the limiting values and conserved energies take finite values. In that case the radial component of velocity at

the point of collision takes a finite nonzero value and can be expanded in the following way, since the second term under the square root is much smaller than the first term,

$$\sqrt{E^2 - f(r) \left(1 + \frac{L^2}{r^2}\right)} = E \left(1 - \frac{f(r)}{2E^2} \left(1 + \frac{L^2}{r^2}\right) + \dots\right) \quad (20)$$

The center of mass energy of collision (18), neglecting the higher order terms can now be written as

$$\begin{aligned} \frac{E_{c.m.}^2}{2m^2} &\approx 1 + (1 - \kappa) \frac{E_1 E_2}{f(r)} - k \frac{|L_1| |L_2|}{r^2} \\ &+ \kappa \left(\frac{E_2}{2E_1} \left(1 + \frac{L_1^2}{r^2}\right) \right) + \kappa \left(\frac{E_1}{2E_2} \left(1 + \frac{L_2^2}{r^2}\right) \right) \end{aligned} \quad (21)$$

In the case where both the particles travel either radially inwards or outwards *i.e.* when $\kappa = 1$, the center of mass energy of collision is given by

$$\frac{E_{c.m.}^2}{2m^2} \approx 1 - k \frac{|L_1| |L_2|}{r^2} + \frac{E_2}{2E_1} \left(1 + \frac{L_1^2}{r^2}\right) + \frac{E_1}{2E_2} \left(1 + \frac{L_2^2}{r^2}\right)$$

which takes a finite value.

In the case where one of the particles under consideration is radially ingoing and the other one is outgoing *i.e.* when $\kappa = -1$ then the center of mass energy of collision to the leading order can be written as

$$\frac{E_{c.m.}^2}{2m^2} \approx \frac{2E_1 E_2}{f(r)} \quad (22)$$

which clearly diverges in the limit $f(r) \rightarrow 0$. Thus the center of mass energy of collision between an ingoing and outgoing particles is arbitrarily large if the metric function $f(r)$ computed at the point of collision is arbitrarily close to zero.

$$\lim_{f(r) \rightarrow 0} E_{c.m.}^2 \rightarrow \infty \quad (23)$$

Any particle satisfying (20) which is initially radially ingoing will necessarily turn back as an outgoing particle at an intermediate value of the radial coordinate between the point of collision and the center if it has a nonzero angular momentum. Whereas if the angular momentum is zero, it will turn back as an outgoing particle if $E < 1$. If $E > 1$ it will pass through the center and then emerge as an outgoing particle. Thus the origin of the outgoing particles which participate in the collisions is easy to explain. The ingoing particles either turn back or pass through the center and emerge as the outgoing particles when their angular momentum for any given value of energy lies in the range (20).

Case 2

We now turn to the second case where the angular momenta of both the particles are arbitrarily close to the extreme values and the conserved energies take finite values. The radial velocity of the particles is extremely

small at the point of collision. Angular momenta of the particles take a value which is given by

$$\begin{aligned} L_1 &\approx \pm r \sqrt{\frac{E_1^2}{f(r)} - 1} \\ L_2 &\approx \pm r \sqrt{\frac{E_2^2}{f(r)} - 1} \end{aligned} \quad (24)$$

Since we are dealing with the case where $f(r) \rightarrow 0$, the angular momenta take an extremely large value $L_1, L_2 \rightarrow \infty$. Thus this situation is not generic and would occur only under exceptional circumstances. We assume that the angular momenta are fine-tuned to the critical values to such an extent that the contribution of the radial component of velocities to the center of mass energy can be ignored.

$$\sqrt{E_1^2 - f(r) \left(1 + \frac{L_1^2}{r^2}\right)} \sqrt{E_2^2 - f(r) \left(1 + \frac{L_2^2}{r^2}\right)} \ll f(r) \quad (25)$$

Thus it follows from (18), (25), (25) that the center of mass energy of collision in this case is given by

$$\frac{E_{c.m.}^2}{2m^2} \approx 1 + \frac{E_1 E_2}{f(r)} (1 - k) + \frac{k}{2} \left(\frac{E_1}{E_2} + \frac{E_2}{E_1} \right) \quad (26)$$

If the angular momenta of two particles are of the same sign, *i.e.* if $k = 1$, then the center of mass energy of collision turns out to be finite

$$\frac{E_{c.m.}^2}{2m^2} \approx 1 + \frac{1}{2} \left(\frac{E_1}{E_2} + \frac{E_2}{E_1} \right) \quad (27)$$

Whereas in the case where the orientation of the angular momenta of the particles is opposite, *i.e.* when $k = -1$ the center of mass energy of collision is given by

$$\frac{E_{c.m.}^2}{2m^2} \approx \frac{2E_1 E_2}{f(r)} \quad (28)$$

which diverges in the limit $f(r) \rightarrow 0$. Thus the center of mass energy of collision between the particles which have nearly zero radial velocity, but angular momenta with the opposite signs is arbitrarily large in the limit where $f(r) \rightarrow 0$ at the point of collision.

$$\lim_{f(r) \rightarrow 0} E_{c.m.}^2 \rightarrow \infty \quad (29)$$

Case 3

We now discuss the third case where the angular momentum of one of the particles is arbitrarily close to the extreme value of the allowed range of the angular momenta. The angular momentum of the second particle is well within the allowed interval.

$$L_1 \approx \pm r \sqrt{\frac{E_1^2}{f(r)} - 1} \quad (30)$$

Both the particles have finite values of the conserved energy. The radial velocity of the first particle takes a vanishingly small value, whereas the radial velocity of the second particle takes a finite value at the point of collision. This situation is again not very generic. The radial velocity of the first particle is assumed to be so small that we can ignore its contribution to the center of mass energy of collision.

$$\sqrt{E_1^2 - f(r) \left(1 + \frac{L_1^2}{r^2}\right)} \sqrt{E_2^2 - f(r) \left(1 + \frac{L_2^2}{r^2}\right)} \ll f(r) \quad (31)$$

Thus from (18),(30),(31), the center of mass energy of collision to the leading order is given by,

$$\frac{E_{c.m.}^2}{2m^2} \approx 1 + \frac{E_1 E_2}{f(r)} - k \frac{L_2 E_1}{r \sqrt{f(r)}} \quad (32)$$

In the limit where at the point of collision $f(r) \rightarrow 0$ we have,

$$\frac{E_{c.m.}^2}{2m^2} \approx \frac{E_1 E_2}{f(r)} \quad (33)$$

which is clearly divergent. Thus the center of mass energy of collision between the particles, one of which has a vanishingly small radial velocity and the other one has a finite radial component of velocity, diverges in the limit where $f(r) \rightarrow 0$.

$$\lim_{f(r) \rightarrow 0} E_{c.m.}^2 \rightarrow \infty \quad (34)$$

The center of mass energy of collision is smaller by the factor of 2 as compared to the first two cases.

Case 4

So far we have dealt with the situations where both the particles have atleast one of the radial or angular component of velocity nonzero. We now turn to the fourth case where one of the colliding particles has vanishing radial as well as the angular components of velocity. From (13),(14) it can be shown that

$$L_1 \approx 0 \quad E_1 \approx \sqrt{f(r)} \approx 0 \quad (35)$$

Thus both the angular momentum and the conserved energy of the particle are approximately zero. Second particle has a finite conserved energy. It has atleast one of the radial or angular component nonzero.

The center of mass energy of collision between two such particles, using (18),(35), to the leading order is given by

$$\frac{E_{c.m.}^2}{2m^2} \approx \frac{E_2}{\sqrt{f(r)}} \quad (36)$$

which clearly diverges in the limit where $f(r) \rightarrow 0$.

$$\lim_{f(r) \rightarrow 0} E_{c.m.}^2 \rightarrow \infty \quad (37)$$

The divergence of the center of mass energy of collision when one of the particles is at rest is however significantly slower as compared to first three cases when both the particles are in motion at the collision.

We have demonstrated in the above the divergence of the center of mass energy of collision between the particles away from the center at the radial location where $f(r) \rightarrow 0$ in various cases.

The situation where both the particles have finite radial velocities at the center is generic and would be of great interest from the perspective of real physical scenarios. Whereas the other situations are not generic since they require finetuning of the angular momentum and energy of the particles as we discussed earlier.

We have shown that, in order to avoid the horizon we must have $f(r) > 0$, whereas to have ultrahigh energy collisions it is necessary that $f(r) \rightarrow 0$. Also at infinity and at the regular center we have $f(0) = f(r \rightarrow \infty) = 1$. Therefore it is necessary for the metric function $f(r)$ to admit a minimum at an intermediate value of the radial coordinate, with minimum value arbitrarily close to zero, nevertheless positive,

$$f'(r) = 0; f(r) \rightarrow 0^+ \quad (38)$$

For a static spherically symmetric asymptotically flat spacetime considered here, the conditions imposed on $f(r)$, namely (3),(10),and (38), that are required for avoidance of the singularity and horizon, and in order to have ultrahigh energy collisions, can be translated to the conditions to be imposed on the timelike Killing vector $k = \partial_t$ making use of (8), and thus these can be written in a coordinate independent way as follows:

I. In order to avoid the horizon, the timelike Killing vector must retain its timelike nature

$$k.k < 0 \quad (39)$$

This is also the condition to avoid singularity away from the center.

II. In order to avoid the singularity we must have

$$\nabla_\mu(k.k) = 0; k.k = -1 \quad (40)$$

at the center.

III. In order to have ultrahigh energy collisions away from the center we must have

$$\nabla_\mu(k.k) = 0; k.k \rightarrow 0^- \quad (41)$$

That is, the norm of the timelike Killing vector must admit a maximum, with the negative value at the maximum which is arbitrarily close to zero.

Here we would like to note that since $f(r)$ admits a minimum if the high energy collisions occur, it implies that the gravity is repulsive in the region below the radius at which the minimum occurs, upto the next maximum as we go inside and $f(r)$ could in principle admit several minima. If it admits only one minimum as in the case of the example we shall discuss in the next section,

gravity is repulsive from the center to the minimum and it is attractive from the minimum to infinity. If $f(r)$ admits several minima with extremely small minimum values then the high energy collisions could occur at several locations in the spacetime. Gravity is repulsive between a maxima and minima and attractive between a minima and maxima as we move from inside out.

It is also worthwhile to note that if not only the first derivative but several derivatives of $f(r)$ take a zero value for some value of $r = r_a$, *i.e.* $f^1(r_a) = f^2(r_a) = \dots = f^n(r_a) = 0$ for large n , where $f^k(r)$ stands for the k^{th} derivative with respect to r , then the value of $f(r)$ could be extremely small in a finite interval around $r = r_a$, where high energy collisions can take place.

III. ENERGY CONDITIONS

In this section we discuss the properties of the matter that would be required to source a regular spacetime geometry without an event horizon or naked singularity, which can host ultrahigh energy collisions. Specifically, we investigate whether matter violates the energy conditions.

The energy momentum tensor for the metric (1) is given by

$$T_\mu^\nu = \frac{1}{8\pi} G_\mu^\nu = \frac{1}{8\pi} \text{Diag} \left[\frac{-1+f+rf'}{r^2}, \frac{-1+f+rf'}{r^2}, \frac{f'}{2} + \frac{f''}{2}, \frac{f'}{2} + \frac{f''}{2} \right] = -\frac{1}{8\pi} \text{Diag} \left[\frac{2M'}{r^2}, \frac{2M'}{r^2}, \frac{M''}{r}, \frac{M''}{r} \right] \quad (42)$$

The density and pressures can be read from the expression above.

$$\rho = -P_r = \frac{1}{8\pi} \frac{(1-f-rf')}{r^2} = \frac{1}{8\pi} \frac{2M'}{r^2} \quad (43)$$

$$P_\theta = P_\phi = \frac{1}{8\pi} \left(\frac{f'}{2} + \frac{f''}{2} \right) = -\frac{1}{8\pi} \frac{M''}{r} \quad (44)$$

The weak energy condition is satisfied if

$$\rho \geq 0 \quad (45)$$

$$(\rho + P_r) \geq 0, (\rho + P_\theta) \geq 0, (\rho + P_\phi) \geq 0 \quad (46)$$

whereas the strong condition is satisfied when

$$(\rho + P_r + P_\theta + P_\phi) \geq 0 \quad (47)$$

as well as (46) hold good.

From (43),(44) we get

$$(\rho + P_r) = 0 \quad (48)$$

$$(\rho + P_\theta) = (\rho + P_\phi) = \frac{2M' - rM''}{r^2} = \frac{1-f}{r^2} + \frac{f''}{2} \quad (49)$$

$$(\rho + P_r + P_\theta + P_\phi) = -\frac{2M''}{r} = \frac{2f'}{r} + f'' \quad (50)$$

As we have shown in the previous section the metric near the center is given by (9) and the mass function to leading order is given by $M(r) = M_3 r^3 + M_4 r^4 + \dots$. It follows from (45) that for the energy density to be positive at the center we must have

$$M_3 > 0 \quad (51)$$

From (45) we get

$$M_4 = \pi \frac{d\rho}{dr} \Big|_{r=0} \quad (52)$$

If the energy density goes on decreasing as we go away from the center, which is reasonable to demand, we must have

$$M_4 < 0 \quad (53)$$

Near the center to the leading order we get

$$(\rho + P_r + P_\theta + P_\phi) = -12M_3 < 0 \quad (54)$$

which implies that the strong energy condition is violated. Whereas

$$(\rho + P_\theta) = (\rho + P_\phi) = -4M_4 r > 0 \quad (55)$$

Thus the weak energy condition is satisfied.

Therefore if we assume that the density is positive and decreases away from the center, the weak energy condition is satisfied but strong energy condition is violated in a region close to the regular center. In this sense, the spacetime contains a ball of dark energy in the region around the center.

At the point of collision, as we showed in the previous section $f = \epsilon \rightarrow 0^+, f' = 0$ and $f'' > 0$. Thus it is clear from the (49),(50) that

$$(\rho + P_\theta) = \frac{1-\epsilon}{r^2} + \frac{f''}{2} > 0 \quad (56)$$

and

$$(\rho + P_r + P_\theta + P_\phi) = f'' \geq 0 \quad (57)$$

Thus both the weak energy condition and strong energy condition are satisfied at the point of collision.

We showed that the conditions for having a regular center and ultrahigh energy collisions in the spacetime imply that the regular center is surrounded by the dark energy fluid that violates the strong energy conditions and respects the weak energy conditions, whereas both the energy conditions are respected at the point of collision.

IV. AN EXAMPLE: THE BARDEEN SPACETIME

In the previous section we described, under what conditions, ultrahigh energy collisions can take place in a class of spherically symmetric static spacetimes containing neither blackholes nor naked singularities. In this section we now present a concrete example to illustrate this point.

We consider here the spacetime metric given by Bardeen [11]. The metric can be written as

$$ds^2 = - \left(1 - \frac{2mr^2}{(r^2 + q^2)^{\frac{3}{2}}} \right) dt^2 + \left(1 - \frac{2mr^2}{(r^2 + q^2)^{\frac{3}{2}}} \right)^{-1} dr^2 + r^2 d\Omega^2 \quad (58)$$

This metric takes the same form as (1) with

$$f(r) = \left(1 - \frac{2mr^2}{(r^2 + q^2)^{\frac{3}{2}}} \right) \quad (59)$$

and when compared with (38) we get

$$M(r) = \frac{mr^3}{(r^2 + q^2)^{\frac{3}{2}}} \quad (60)$$

There are two parameters in the solution, namely m and q . Here m can be interpreted as the mass parameter for the system. The other parameter q can be interpreted as the magnetic charge, as it was shown in [12] that this metric solves Einstein equations with a self-gravitating nonlinear magnetic monopole as source. Various other examples of the regular blackholes have also been discovered and studied recently [14].

First of all $M(r)$ can be expanded in the following way near the center $r = 0$.

$$M(r) = \frac{m}{q^3} r^3 - \frac{3m}{2q^5} r^5 + \dots \quad (61)$$

This implies that $M_0 = M_1 = M_2 = 0$. Thus it follows from (8) that the center is not singular. Also, the spacetime is asymptotically flat. We note that the function $f(r)$ admits only one minimum at

$$r_{min} = \sqrt{2}q \quad (62)$$

and the minimum value is given by

$$f(r_{min}) = 1 - \frac{4}{3^{\frac{3}{2}}} \frac{m}{q} \quad (63)$$

When $m > \frac{3^{\frac{3}{2}}}{4}q$ the minimum value is less than zero, $f(r_{min}) < 0$. Since $f(r = 0) = f(r \rightarrow \infty) = 1 > 0$, the function $f(r)$ must admit a zero for two values of r , $f(r) = 0$ for $r = r_1, r = r_2$, where $0 < r_1 < r_{min}$ and $r_{min} < r_2 < \infty$. The Bardeen metric in this case corresponds to the blackhole with inner and outer horizons located at $r = r_1$ and $r = r_2$ respectively. The blackhole does not have a singularity at the center and this was the first example of a nonsingular blackhole.

When $m = \frac{3^{\frac{3}{2}}}{4}q$, the minimum value of the function is zero, $f(r_{min}) = 0$, and both the zeros coincide with the minimum $r_{min} = r_1 = r_2 = 0$. This corresponds to the case where the inner and outer event horizons coincide. In such a case, the Bardeen solution corresponds to an extremal blackhole with a regular center.

When $m < \frac{3^{\frac{3}{2}}}{4}q$, the minimum of the function is greater than the zero at $f(r_{min}) = 0$ and thus $f(r)$ is strictly positive everywhere in the spacetime. There is then no event horizon or singularity in the spacetime.

We are interested in the case where the mass parameter is smaller than the value required for the minimum of $f(r)$ to be zero,

$$m = \frac{3^{\frac{3}{2}}}{4}q(1 - \epsilon) \quad (64)$$

where $0 < \epsilon < 1$. The minimum value of the function $f(r)$ at $r = q$ is given by

$$f(r_{min}) = \epsilon \quad (65)$$

As discussed before, at the center and at infinity this function is unity and it admits only one extremum, namely minimum at $r = q$. Thus it is everywhere positive. The solution is regular and admits no horizons.

We now describe high energy particle collisions. For the sake of simplicity and definitiveness we restrict our attention to two particles that move along radial direction along the same line. The angular momenta of these particles are zero $L_1 = L_2 = 0$. We further assume that $E_1 = E_2 = E$. The radial component of velocity from (14) in this case is given by

$$U^r = \pm \sqrt{E^2 - f(r)} \quad (66)$$

It follows that we must have $E \geq \sqrt{\epsilon}$. If $E = \sqrt{\epsilon}$ then particle stays at rest at $r = r_{min} = \sqrt{2}q$. If $\sqrt{\epsilon} \leq E < 1$ then an initially ingoing particle will turn back at a radial coordinate between the center and the minimum and emerge as an outgoing particle. This implies that the gravity is "repulsive" close to the center. This is quite surprising that gravity can be repulsive in the absence of the naked singularities. If the particle has energy $E > 1$ then initially ingoing particle passes through the center and emerges as an outgoing particle from the other side, whereas if $E = 1$ the ingoing particle asymptotically approaches the center.

We can now take two particles each with $E > \sqrt{\epsilon}$ and $E \neq 1$. An initially ingoing particle will emerge as an

outgoing particle either after getting reflected back or after passing through the center. The center of mass energy of collision between the ingoing and outgoing particles, using (18) is given by

$$\frac{E_{c.m.}^2}{2m^2} = \frac{2E^2}{f(r)} \quad (67)$$

The center of mass energy is maximum where $f(r)$ is minimum. From (18) we get,

$$\frac{E_{c.m.}^2}{2m^2} = \frac{2E^2}{\epsilon} \quad (68)$$

The center of mass energy of collision can be arbitrarily large in the "near-extremal" limit where $\epsilon \rightarrow 0$

$$\lim_{\epsilon \rightarrow 0} \frac{E_{c.m.}^2}{2m^2} = \frac{2E^2}{\epsilon} \rightarrow \infty \quad (69)$$

We now consider a collision between a particle which is at rest at $r = r_{min}$, i.e the particle with energy $E_1 = \sqrt{\epsilon}$ and zero angular momentum, with another radially ingoing particle with energy E_2 which has a finite radial velocity. It follows from (18) that the center of mass energy of collision between the particles is given by

$$\frac{E_{c.m.}^2}{2m^2} = 1 + \frac{E_2}{\sqrt{\epsilon}} \quad (70)$$

This clearly diverges in the limit $\epsilon \rightarrow 0$,

$$\lim_{\epsilon \rightarrow 0} \frac{E_{c.m.}^2}{2m^2} \approx \frac{E_2}{\sqrt{\epsilon}} \rightarrow \infty \quad (71)$$

We plot the quantities $(\rho + P_\theta)$ and $(\rho + P_r + P_\theta + P_\phi)$ as a function of radius in Fig1. It can be clearly seen that the weak energy condition is respected everywhere, whereas the strong energy condition is violated in the region near center upto a certain radius below the place where collision takes place. The violation in the regular blackholes was discussed in [15]

Thus we have demonstrated here the ultrahigh energy collisions of the particles in an explicit example of a spacetime geometry which contains no event horizons or a naked singularity.

V. CONCLUSIONS

We showed here that it would be possible to have ultrahigh energy particle collisions in a spacetime which does not contain any event horizon or a naked singularity. Previously, the phenomenon of ultrahigh energy collisions had been explored in the blackhole geometries. We extended the result to geometries containing naked singularities, and further we find here that it is not necessary to have horizon or singularities for high energy collisions to occur.

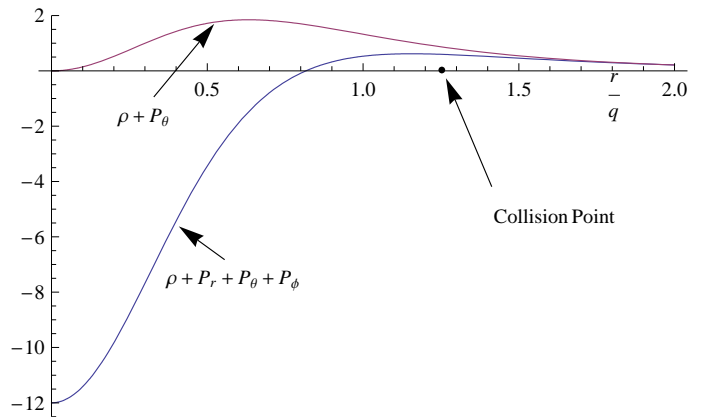


FIG. 1: In this figure we plot $(\rho + P_\theta)$ and $(\rho + P_r + P_\theta + P_\phi)$ as a function of $x = \frac{r}{q}$ in units of $\frac{m}{4\pi q^3}$. The weak energy condition is satisfied everywhere. Whereas the strong energy condition is violated in a ball around the center with the radius smaller than the location of the collision. It is respected everywhere else.

We obtained the conditions to be imposed on the general spherically symmetric static spacetime in order to avoid the horizon and singularities and in order to have ultrahigh energy collisions. We showed that it order to avoid the horizon the norm of the timelike Killing vector $k = \partial_t$ should remain negative everywhere. This condition also ensures that there is no singularity away from the center. In order to avoid the singularity at the center, the norm of the Killing vector should admit a minimum and must take a value -1 . In order to have ultrahigh energy collisions between the particles it is necessary for the norm of the timelike Killing vector to admit a maximum away from the center with the value very close to zero. If these conditions are met then the center of mass energy between the radially ingoing and outgoing particles (with finite radial component of velocity, which is the situation that would occur generically), is arbitrarily large depending on how close to zero is the norm of the timelike Killing vector. This is the condition for the existence of the high blueshift surface for the approaching particles and high redshift surface for the receding particles.

The conditions for having a regular center and ultrahigh energy collisions in the spacetime imply that the strong energy condition is violated in the region surrounding the center, whereas the weak energy condition is respected. Thus the region near the regular center is filled with dark energy fluid. Both the energy conditions are respected at the location where high energy collisions take place.

We demonstrate this point here with a concrete example of the Bardeen spacetime which corresponds to a perfectly regular geometry without any horizon or singularities for a certain parameter range. We show that the conditions mentioned above are met with and it is possi-

ble to have ultrahigh energy collisions of particles in the Bardeen spacetime for appropriate values of parameters.

The purpose of this paper was to show that unlike the common belief, particle acceleration and ultrahigh energy collisions can take place in a perfectly regular spacetime without blackholes or naked singularities. Some of the energy conditions must be, however, violated by the matter fields in such a case near the center. In other words, in a fully energy conditions preserving spacetime, either a blackhole or a naked singularity becomes a necessary condition for the high energy collisions to take place. In a future work we would like to generalize these results to

more general spacetime geometries.

VI. ACKNOWLEDGEMENTS

We would like to thank K. Nakao and M. Kimura for discussions and going through the manuscript carefully during their recent visit to TIFR. MP would like to thank T. Harada for interesting discussion during the ICGC meeting in Goa.

-
- [1] M.Banados, J.Silk, S.M.West, Phys. Rev. Lett. 103, 111102 (2009).
 - [2] E.Berti, V.Cardoso, L.Gualtieri, F.Pretorius, U.Sperhake, Phys. Rev. Lett. 103, 239001 (2009); Jacobson T. Jacobson, T.P.Sotiriou, Phys. Rev. Lett. 104, 021101 (2010).
 - [3] T. Harada, M.Kimura, Phys. Rev. D 84, 124032(2011); Phys. Rev. D 83, 024002(2011); Phys. Rev. D 83, 084041(2011); A. Grib, Y.Pavlov, Grav. Cosmol. 17, 42, (2011), arXiv:1007.3222 (2010); arXiv:1004.0913(2010), M. Bejger, T. Piran, M. Abramowicz, F. Hkanson arXiv:1205.4350 (2012).
 - [4] M.Banados, B.Hassanain, J.Silk, S.West, Phys. Rev. D 83, 023004,(2011); A. Williams, Phys. Rev. D 83, 123004,(2011).
 - [5] M.Kimura, K.Nakao, H.Tagoshi, Phys. Rev. D 83, 044013,(2011).
 - [6] T. Igata, T.Harada, M. Kimura, arXiv:1202.4859 (2012); J. Yang, Y. Li, Y. Li, S. Wei, Y.Liu, arXiv:1202.4159 (2012)[hep-th]; [hep-th] V. Frolov, arXiv:1110.6274 (2011); O. B. Zaslavskii, Phys. Rev. D 85 (2012) 024029; arXiv:1105.0303 (2011); Phys. Rev. D 84, 024007(2011); JETP Lett. 92, 571 (2010); arXiv:1107.3964 (2011); Class. Quant. Grav. 28, 105010, (2011); Phys. Rev. D 82, 083004,(2010). C. Zhong, S. Gao, JETP Lett. 94, 8,589 (2011); J. Sadeghi, B. Pourhassan, arXiv:1108.4530 (2011); Y. Zhu, S. Wu, Y. Jiang, G. Hong Yang, Phys. Rev. D 84, 123002 (2011); A. Abdujabbarov, B. Ahmedov, B. Ahmedov, Phys. Rev. D, 84, 4, 044044 (2011); S. Gao, C. Zhong, Phys. Rev. D 84, 044006,(2011); W. Yao, S. Chen, C. Liu, J. Jing, Eur. Phys. J. C 72, 1898 (2012); J.Said, K. Adami, Phys. Rev. D 83, 104047,(2011); A. Grib, Y. Pavlov, O. Piattella, arXiv:1105.1540 (2011); C. Liu, S. Chen, J. Jing, arXiv:1104.3225 (2011); Y. Zhu, S. Wu, Y. Xiao Liu, Y. Jiang, Phys. Rev. D 84, 043006,(2011); C. Liu, S. Chen, C. Ding, J. Jing, Phys. Lett. B 701, 285, (2011); Y. Li, J. Yang, Y. Li, S. Wei, Y. Liu, Class. Quantum Grav. 28 225006 (2011); R. Plyatsko, O. Stefanyshyn, M. Fenyk, Phys. Rev. D 82, 044015 (2010); P. Mao, R. Li, L. Jia, J. Ren, arXiv:1008.2660 (2010); S. W. Wei, Y. X. Liu, H. T. Li, and F. W. Chen, JHEP 1012, 066 (2010); S. W. Wei, Y. X. Liu, H. Guo, and C. E. Fu, Phys. Rev. D 82, 103005 (2010).
 - [7] M. Patil, P. Joshi, Class. Quantum Grav. 28, 235012 (2011); Phys. Rev. D 84, 104001 (2011).
 - [8] M. Patil, P. Joshi, K. Nakao, M. Kimura arXiv:1108.0288(2011).
 - [9] M. Patil, P. Joshi, arXiv:1106.5402(2011).
 - [10] M. Patil, P. Joshi, arXiv:1112.2525(2011).
 - [11] J. Bardeen, presented at GR5, Tiflis, U.S.S.R., and published in the conference proceedings in the U.S.S.R., 1968
 - [12] E.Ayon-Beato, A. Garcia, Phys. Lett. B 493 149 (2000)
 - [13] P. Mazur, E. Mottola arXiv:gr-qc/0109035 (2001); M. Visser, D.Wiltshire, Class.Quant.Grav. 21 (2004) 1135-1152; C. Ghezzi, Astrophys.Space Sci.333:437-447,(2011); R. Chan, M. Silva, P. Rocha, A. Wang, JCAP 0903:010,(2009); JCAP 0811:010,2008; J. Lemos, O. Zaslavskii, Phys.Rev.D78:024040,(2008).
 - [14] E.Ayon-Beato, A. Garca, Phys.Rev.Lett.80:5056-5059,(1998); Gen.Rel.Grav. 31 (1999) 629-633; Phys.Lett. B464 (1999) 25.
 - [15] O. B. Zaslavskii, Physics Letters B 688 (2010) 278-280